Reading course: Banach spaces and algebras (MATH5002)

This document will list the parts of the book we have read. When I say something is "important", or just list it without comment, then it might very well be on an exam. If something is "interesting", then I think it provides useful context, but an exam wouldn't ask you to state and prove such a result.

My view is that the book has turned out to be harder to read than I expected, and so we will in the end not study quite as much as I had originally expected to.

1 Revision of normed spaces; dual spaces; Hahn-Banach

See Sections 2.1, 2.2, 2.3, 2.5, 3.1. All of this should be revision from Linear Analysis I, and hence is not examinable (but will be freely used in the rest of the course).

2 Weak and weak*-topologies; second duals; geometric forms of Hahn-Banach; Krein-Milman

- Theorem 3.1, the Hahn-Banach theorem stated for sub-linear functionals, is probably new (but essentially the same as versions of Hahn-Banach as seen before).

- Section 3.3; don't worry too much about proofs. Read section 3.4.

- Section 3.5: Theorem 3.18 and Theorem 3.21 (Banach-Alaoglu).

- Section 3.7 (these are often also called the "geometric forms of Hahn-Banach"). Theorem 3.26 and all its corollaries.

- Section 3.8: Theorem 3.31.

3 Baire category, Open Mapping, Closed Graph, Uniform boundedness theorems

- Section 3.9: Theorem 3.34 and Theorem 3.36. You need to read the relevant part of Chapter 1 as well.

- Section 3.10: Lemma 3.38

- Section 3.11: Theorem 3.40 (also remember my "correction" to this.) Try to understand carefully how this fits with Lemma 3.38. Understand the corollaries. Theorem 3.45

- Section 3.12 is routine, but used later.

- Sections 3.15 and 3.16 are interesting, but not examinable.

4 Basics of Banach algebras; constructions; group of units

- Section 4.1, Section 4.2. Nothing really to learn here, but important background.

- Section 4.3: Standard constructions; lemma 4.8 isn't so important.

- Section 4.4: Lemma 4.10 is very useful. It's corollaries are important too.

5 Spectrum; Characters; Gelfand Theory

- Section 4.5: Theorem 4.17, Theorem 4.19, Lemma 4.22. Don't worry about the proof of Theorem 4.23, but know the statement.

- Section 4.6 is interesting, Theorem 4.28 is useful in calculations.

- Section 4.9: Theorem 4.36 and its corollaries. Proposition 4.39 is interesting.

- Section 4.10: Theorem 4.43.

- Section 4.11: Theorem 4.46 and the corollary. Understand examples 4.49 and 4.50 (but 4.52 and 4.53 are off-topic). Theorem 4.54 is important, as is Theorem 4.59 (don't worry about 4.55–4.58). Example 4.62 and Theorem 4.63 are the classical applications of this theory.

6 Commutative Banach algebras; holomorphic functional calculus

- Section 4.14: This is really just background, and is non-examinable.

- Section 4.15: The statement of Theorem 4.89 is very important, but don't worry about the proof. Corollary 4.90 and Lemma 4.91. Propositions 4.93 and 4.94. Theorem 4.95. Proposition 4.98. Theorem 4.100 is a typical application of this material.

- Section 4.16: Non-examinable, but useful background.

7 C*-algebras; continuous functional calculus

- Sections 2.13 and 2.14. These should be (mostly) revision from past courses. But make sure you know the material.

- Section 2.15 is useful background material.

- Section 4.7: all three theorems are important to know, but the proofs are non-examinable.

- Section 6.1. General background; make sure you understand the examples. Ignore Lemma 6.3 and Theorem 6.4.

- Sections 6.2 and 6.3 are a little off-topic. We shall skip them on a first reading, but material here will be needed later, so we'll come back if, and only if, we need to.

- Section 6.4: Skip Corollary 6.18, but everything else is important. Our aim here is Theorem 6.24.

- Section 6.5: Theorem 6.26 and corollaries. Skip long discussion on page 274. Proposition 6.29, Theorem 6.30, Proposition 6.32. The remaining results are nice background only.

- Section 6.6: Theorem 6.41 (and have some understanding of how the previous results combine to prove this). Theorem 6.47. I don't like the way this is proved– so the proofs here are not examinable; but the statements should be understood.

8   Representation theory; modules; radicals; uniqueness of norm

- Section 5.1: Useful background.

- Section 5.3: The aim here is Theorem 5.9 and Corollary 5.10. Ignore the later material.

- Section "Automatic continuity" starting on page 241. Theorem 5.25 and Corollary 5.26. I am not sure I like these proofs, so I might come up with some of my own. Otherwise we'll also need to go back are read Section 3.13.

9   Applications and examples to group algebras

We probably will have run out of time by this point. Nothing here will be examinable; I will come up with the reading if needs be.