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On higher-dimensional amenability of Banach algebras

ABSTRACT. For any $n \geq 1$, a Banach algebra \mathcal{A} is called *n-amenable* if the continuous Hochschild cohomology $\mathcal{H}^n(\mathcal{A}, X^*) = \{0\}$ for every Banach \mathcal{A} -bimodule X . It is clear that \mathcal{A} is *n-amenable* but not $(n - 1)$ -amenable if and only if the *weak bidimension* of \mathcal{A}

$$db_w \mathcal{A} = \inf \{n : \mathcal{H}^{n+1}(\mathcal{A}, X^*) = \{0\} \text{ for all Banach } \mathcal{A}\text{-bimodule } X\}$$

is equal to $(n - 1)$. Other equivalent definitions for higher-dimensional amenability of \mathcal{A} will be given. Connections between higher-dimensional amenability of \mathcal{A} and closed ideals with bounded approximate identities (b.a.i.) will be considered. We will show that the weak bidimension db_w of the tensor product $\mathcal{A} \widehat{\otimes} \mathcal{B}$ of Banach algebras \mathcal{A} and \mathcal{B} with b.a.i. satisfies

$$db_w \mathcal{A} \widehat{\otimes} \mathcal{B} = db_w \mathcal{A} + db_w \mathcal{B}.$$

We prove further that the formula does *not* hold for algebras with no b.a.i, nor for algebras with only 1-sided b.a.i. The well-known trick of adjoining an identity to the algebra does not work for the tensor product of algebras.

References: Lykova, Z.A. The higher-dimensional amenability of tensor products of Banach algebras, accepted for publication in *J. Operator Theory* in June 2009, 24 pp. (arXiv:0904.4548v1 [math.KT] 29 Apr 2009.)