Further reading for mini-course "Locally compact quantum groups"

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In May 2014 I gave a series of four lectures on the topic of Locally compact quantum groups, as part of the subsection "Operator Spaces, Locally Compact Quantum Groups and Amenability" as part of the "Thematic Program on Abstract Harmonic Analysis, Banach and Operator Algebras". I looked at the Banach algebras $L^1(G), A(G)$ from an operator algebraic perspective, and then gave some details about compact quantum groups, before quickly talking about the locally compact case and some recent work on Banach algebraic cohomology.

This document is an annotated bibliography, giving some further reading. In my opinion, much of the original literature is very readable, although there is perhaps no single starting point. This list is very, very far from complete, and in particular says very little about the large amount of recent work, especially in the compact case. Instead, I try to give details on what to read to understand the basics— then you can explore the wider literature for yourself!

(Comments very welcome.)

1 Books

To my knowledge, there is no textbook which is close to my presentation (namely, focused very much on the operator algebraic perspective).

- T. Timmermann, An invitation to quantum groups and duality. (European Mathematical Society (EMS), Zürich, 2008). MR2397671.

  [This is a readable introduction to Hopf algebras which specialises to compact quantum groups from the algebraic perspective. Then the theory is re-introduced from the C^*-algebra perspective, but making use of the already discussed algebraic theory. There is a brief introduction to the locally compact case, and then discussion of groupoids.]

- A. Klimyk, K. Schmüdgen, Quantum groups and their representations. (Springer-Verlag, Berlin, 1997). MR1492989.

  [A very algebraic book, but there is a lot of material on compact quantum groups, concentrating on the corepresentation theory of deformed Lie groups.]

- S. Neshveyev, L. Tuset, "Compact Quantum Groups and Their Representation Categories", draft currently available at `http://folk.uio.no/sergeyn/research.html`

  [A more advanced book— the first chapter is a good introduction to compact quantum groups, and the rest of the book details modern research in the area.]

- E. C. Lance, Hilbert C^*-modules: A toolkit for operator algebraists. (Cambridge University Press, Cambridge, 1995). MR1325694.

  [A good resource if you are interested in multiplier algebras, and related ideas. One chapter looks a quantum groups.]

2 Surverys

These are various survery articles, listed vaguely in the order I would attack them.

- J. Kustermans, "Locally compact quantum groups" in *Quantum independent increment processes. I*, Lecture Notes in Math. 1865, pp. 99–180 (Springer, Berlin, 2005). MR2132094. (Available via an internet search).

  [An excellent introduction to the subject, from a summer school which Kustermans gave. Makes efforts to quickly introduce the needed operator algebra theory, and then gives the basics of the locally compact case, without proofs (but with various useful exercises). Ends with examples and an extensive bibliography.]

- J. Kustermans, L. Tuset, "A survey of C^*-algebraic quantum groups. I", *Irish Math. Soc. Bull.* 43 (1999) 8–63. MR1741102.

  Part II same journal, 44 (2000) 6–54. MR1765697.

  [Two surveys, saying more about the algebraic and compact side of the theory. Interesting from a historical perspective, and for links between this theory and the pure algebra concept of a "quantum group".]

3 Compact quantum groups

- S. L. Woronowicz, "Compact quantum groups", in *Symétries quantiques (Les Houches, 1995)* 845–884 (North-Holland, Amsterdam, 1998). MR1616348.

  [The original paper, detailing the theory I presented in my talks. Can be hard to get hold of, and maybe hard to read in isolation.]

- A. Maes, A. van Daele, "Notes on compact quantum groups", *Nieuw Arch. Wisk. (4)* 16 (1998) 73–112. MR1645264.

  [See also arXiv:math/9803122. A self-contained survey detailing the same theory.]

- S. L. Woronowicz, "Compact matrix pseudogroups", *Comm. Math. Phys.* 111 (1987) 613–665. MR901157.

  [This details an earlier axiomatisation of the theory, where the algebra is generated by a single corepresentation, and we assume some sort of Hopf algebra, but without the cancellation condition. Useful to read to see other aspects of theory, and because many examples actually more easily fit into this older framework. Also read. . .]

- S. L. Woronowicz, "A remark on compact matrix quantum groups", *Lett. Math. Phys.* 21 (1991) 35–39. MR1088408.

  [Simplifies the axioms of the previous paper: the generating corepresentation u need only satisfy that u and $\overline{u}$ are invertible.]

- See also other papers by e.g. Woronowicz, van Daele, S. Wang.

4 Locally compact quantum groups

- J. Kustermans, S. Vaes, "Locally compact quantum groups", *Ann. Sci. École Norm. Sup. (4)* 33 (2000) 837–934. MR1832993

  [The main paper laying out the theory in the C^*-algebra case.]

- J. Kustermans, S. Vaes, "Locally compact quantum groups in the von Neumann algebraic setting", *Math. Scand.* 92 (2003) 68–92. MR1951446.

  [And in the von Neumann setting, though this paper is not self-contained.]

- A. van Daele, "Locally compact quantum groups. A von Neumann algebra approach" arXiv:math/0602212 [math.OA].

  [A self-contained, slightly different, approach to the von Neumann case.]

- J. Kustermans, "Locally compact quantum groups in the universal setting", *Internat. J. Math.* 12 (2001) 289–338. MR1841517.

  [Details how to form the "full" C^*-algebra version, paralleing the passage from $C_r^*(G)$ to $C^*(G)$.]

- T. Masuda, Y. Nakagami, S. L. Woronowicz, "A C^*-algebraic framework for quantum groups", *Internat. J. Math.* 14 (2003) 903–1001.

  [Gives a more complicated, but equivalent, axiomatisation. Well worth reading to gain further intuition on the objects involved. The appendixes are really useful.]

- See also papers (and arXiv preprints) by Kustermans and Kustermans–Vaes which carefully expound on the theory of weights on C^*-algebras, one-parameter groups on C^*-algebras and their extensions to multiplier algebras, and so forth.

5 Multiplicative unitary approach

- S. Baaj, G. Skandalis, "Unitaires multiplicatifs et dualité pour les produits croisés de C^*-algèbres", *Ann. Sci. École Norm. Sup. (4)* 26 (1993) 425–488. MR1235438.

  [Perhaps the first paper to systematically study quantum groups from the perspective of multiplicative unitaries. Many later papers build on ideas in this paper.]

- S. L. Woronowicz, "From multiplicative unitaries to quantum groups", *Internat. J. Math.* 7 (1996) 127–149. MR1369908.

  [Shows that given a multiplicative unitary with some extra structure, one can build a quantum group, with all the expected structure, duality theory etc. Used by some of the later papers detailed above.]

- P. Sołtan, S. L. Woronowicz, "From multiplicative unitaries to quantum groups. II", *J. Funct. Anal.* 252 (2007) 42–67. MR2357350.

  [Follow-on paper, showing that the C^* and von Neumann algebras built in the previous paper are independent of the multiplicative chosen. Also builds a "full" C^*-algebra picture.]